Ratio & Percent

1. Ratios

A ratio is used to make comparisons between two similar terms. The items within a ratio are typically of the same units and the resulting comparison is dimensionless (i.e., no units).

Ratios are typically expressed in one of three ways, the first being the most common:

- A fraction (division): $\frac{5}{6}$
- In words, using "to": 5 to 6
- With a colon: 5:6

For instance, RU has 409 biology majors and 76 math majors. As a ratio the number of biology majors to math majors is

$$\frac{\text{\# biology majors}}{\text{\# math majors}} = \frac{409 \text{ students}}{76 \text{ students}} = 5.38..$$

The number of biology majors : math majors is approximately 5:1.

Ex.) The *Energy Payback Ratio* is used to evaluate power plants:

$$EPR = \frac{energy\ generated}{energy\ consumed}$$

What is the payback ratio when 2,800,000 GJ are consumed to generate 11,350,000 GJ in a plant powered by natural gas?

Use the given equation and substitute the known values (note the result is dimensionless)

$$EPR = \frac{energy\ generated}{energy\ consumed} = \frac{11{,}359{,}000}{2{,}800{,}000} = 4.057 \approx \frac{4}{1}$$

The ratio is about 4:1

Ex.) In an experiment Mendel interbred true yellow round seed peas with true green wrinkled seed peas. The F_2 progeny produced were
315 yellow round seeds, 108 green round seeds, 101 yellow wrinkled seeds and 32 green wrinkled seeds.
Approximate the ratio of phenotypes as A:B:C:D so that A+B+C+D=16

$$315 + 108 + 101 + 32 = 556$$

$$\frac{315}{556} = 0.5665467625899281, \qquad \frac{108}{556} = 0.1942446043165468$$

$$\frac{101}{556} = 0.1816546762589928, \qquad \frac{32}{556} = 0.0575539568345324$$

Multiplying by 16

$$\frac{315}{556} \cdot 16 = 9.064748201438849, \qquad \frac{108}{556} \cdot 16 = 3.107913669064748$$

$$\frac{101}{556} \cdot 16 = 2.906474820143885, \qquad \frac{32}{556} \cdot 16 = 0.920863309352518$$

Estimated the ratios $9:3:3:1$

2. Normalization

Ratios are one way of normalizing data in order to express relative values rather than actual values. One quantity is often normalized by a second quantity to eliminate the second quantity as a variable.

Ex.) Data from 2012 ([1]) indicates the United States' oil consumed 18,490 (thousand) barrels per day. Japan consumed 4,726 (thousand) while Germany consumed 2,388 (thousand). The estimated population of the United States was 313 million, Japan 130 million, and Germany 82 million. What is the ***per capita*** consumption of oil in each of the three countries?

Per capita means "per person." To determine the per capita consumption, take the amount of oil consumed for each country and divide it by the population.

US: $\frac{18{,}490{,}000\frac{barrels}{day}}{313{,}000{,}000\ persons} = 0.059073482428115 \approx 0.06\ barrels\ per\ day\ per\ person$

Japan: $\frac{4{,}726{,}000\frac{barrels}{day}}{130{,}000{,}000\ persons} = 0.0036353846153846 \approx 0.004\ barrels\ per\ day\ per\ person$

Germany: $\frac{2{,}388{,}000\frac{barrels}{day}}{82{,}000{,}000\ persons} = 0.0291219512195122 \approx 0.03\ barrels\ per\ day\ per\ person$

A barrel of oil is defined as 42 gallons (US). This means that in the US each person uses approximately $0.06 \cdot 42 = 2.52$ gallons of oil each day.

3. Percent

A ***percentage*** is a ratio expressed as part of 100 or per hundred. "**Percent**" means "per 100." To calculate a percentage use

$$Percentage = \frac{subgroup}{total} \times 100$$

Ex.) A class has 52 female and 38 male students. What is percentage of female students? What is ratio of female to male students?

The total number of students is $52 + 38 = 90$ students. The subgroup being examined is the number of female students (52).

The percentage of female students is then

$$\frac{52}{90} \times 100 \approx 57.78$$

Therefore, approximately 57.78% of the students are female.

The ratio of female to male students is 52:38, or $\frac{52}{38} = \frac{26}{19}$.

Ex.) It was reported that between 2009-2013 approximately 12.7% of the population of Zambia was infected with HIV ([2]). If the population of Zambia is approximately 14.08 million in 2012, how many individuals are infected with HIV? What is the ratio between healthy (not infected) and infected people?

To calculate the number of people infected with HIV in Zambia, multiply the percentage (as a decimal) times the population:

$$0.127 \cdot 14{,}080{,}000 = 1{,}788{,}160$$

There are 178,816 people infected with HIV in Zambia.

The ratio between healthy and infected is

$$\frac{87.3\ \%}{12.7\ \%} = \frac{873}{127} = 6.874015748031496$$

This means the ratio of healthy to infected is approximately 7:1.

4. Percentage as a Measure of Change

The amount of change with time can also be expressed as a percentage. The percent increase or decrease in a value from one time to the next is given by the formula for ***percentage change***:

$$\frac{\text{final value} - \text{initial value}}{\text{initial value}} \times 100 = \%\ \text{change}$$

Note, this value can be positive – indicating an increase, or negative – indicating a decrease.

Ex.) Over the course of one week, bacteria colonies in a lab increased from 720 colonies per liter to 1260 colonies per liter. What is the percentage change?

The initial amount is 720 colonies/L and the final amount is 1260 colonies/L.

$$\%\ \text{change} = \frac{\text{final value} - \text{initial value}}{\text{initial value}} \times 100 = \frac{1260\ \frac{\text{colonies}}{\text{L}} - 720\ \frac{\text{colonies}}{\text{L}}}{720\ \frac{\text{colonies}}{\text{L}}} \times 100$$
$$= 75\%$$

The percent change is 75%, implying an increase of 75% in the number of colonies per liter.

Ex.) The blood sugar of an individual changes from 4.5 mmol/L (millimol per liter) before lunch to 6 mmol/L after lunch. What is the percentage change?

The initial amount is 4.5mmol/L and the final amount is 6 mmol/L.

$$\%\ \text{change} = \frac{\text{final value} - \text{initial value}}{\text{initial value}} \times 100 = \frac{6\ \frac{\text{mmol}}{\text{L}} - 4.5\ \frac{\text{mmol}}{\text{L}}}{4.5\ \frac{\text{mmol}}{\text{L}}} \times 100 \approx 33.3\%$$

Ex.) The atomic mass of glucose ($C_6H_{12}O_6$) is 180 Da. Thus 1 mol has mass 180 g. Convert 4.5mmol/L and 6 mmol/L to mg/dL (milligram per deciliter), and recalculate the percentage change.

$$4.5\ \frac{\text{mmol}}{\text{L}} = 4.5\ \frac{\text{mmol}}{\text{L}} \cdot \frac{1\ \text{mol}}{1000\ \text{mmol}} \cdot \frac{180\ \text{g}}{1\ \text{mol}} \cdot \frac{1000\ \text{mg}}{1\ \text{g}} \cdot \frac{1\ \text{L}}{10\ \text{dL}} = 81\ \frac{\text{mg}}{\text{dL}}$$

$$6\ \frac{\text{mmol}}{\text{L}} = 6\ \frac{\text{mmol}}{\text{L}} \cdot \frac{1\ \text{mol}}{1000\ \text{mmol}} \cdot \frac{180\ \text{g}}{1\ \text{mol}} \cdot \frac{1000\ \text{mg}}{1\ \text{g}} \cdot \frac{1\ \text{L}}{10\ \text{dL}} = 108\ \frac{\text{mg}}{\text{dL}}$$

$$\%\ \text{change} = \frac{\text{final value} - \text{initial value}}{\text{initial value}} \times 100 = \frac{108\ \frac{\text{mg}}{\text{dL}} - 81\ \frac{\text{mg}}{\text{dL}}}{81\ \frac{\text{mg}}{\text{dL}}} \times 100 \approx 33.3\%$$

5. Percentage Difference and Percentage Error

Similar to the formula to calculate a percentage change, there exists a formula to calculate the ***percentage difference*** between any two comparable values.

$$\%\ \text{difference} = \frac{\text{comparison value} - \text{reference value}}{\text{reference value}} \times 100$$

Ex.) Carbon is stored is various "reservoirs" on Earth: in gases in the atmosphere, in vegetation on land, in ocean waters, and in fossil fuels (such as coal and petroleum). The amount of carbon stored as fossil fuels is estimated to be 3,700 Pg (petagram; 1 Pg = 10^{15} g), whereas the amount of carbon stored in vegetation is estimated to be 2,300 Pg. What is the percentage difference between carbon stored in fossil fuels and carbon stored in vegetation?

$$\%\ \text{difference} = \frac{\text{comparison value} - \text{reference value}}{\text{reference value}} \times 100$$

$$= \frac{2300\ \text{Pg} - 3700\ \text{Pg}}{3700\ \text{Pg}} \times 100$$

$$= -37.83783783783784$$

This means that the amount of carbon in vegetation is approximately 37.8% less than the amount of carbon in fossil fuels.

Quite similar to the percentage difference formula is one that can be used to calculate the ***percentage error*** associated with a measurement. The percentage error is used to determine how close a measurement is to the true or correct value.

$$\%\ \text{error} = \frac{\text{measured value} - \text{true value}}{\text{true value}} \times 100$$

Ex.) ([3])The "General Sherman" sequoia tree growing in Sequoia National Park in California is 83.42 meters tall. Suppose you visit the park and estimate the height of the tree to be 275 ft tall. What is the percentage error of your estimate?

$$275\text{ ft} = 275\text{ ft} \cdot \frac{1\text{ m}}{3.280840\text{ ft}} = 83.81999731776009\text{ m}$$

$$\%\text{ error} = \frac{83.81999731776009\text{ m} - 83.43\text{ m}}{83.43\text{ m}} \times 100$$

$$= 0.467454534052602\%$$

The percentage error is approximately 0.5% (which makes the guess quite good!).

Note, since the percentage error was positive, the measured value is an overestimation of the true value.

Ex.) While star gazing an individual measures the angular diameter of the moon to determine the distance from the Earth to the moon. The calculated distance is 355,700 km. If the actual distance from the Earth to the moon is 384,400 km, then what is the percentage error?

$$\%\text{ error} = \frac{355700\text{ km} - 384400\text{ km}}{384400\text{ km}} \times 100 = -7.466181061394381\%$$

The star gazer underestimated the distance from the Earth to the moon by approximately 7.5%.

References:
(1) http://www.eia.gov/countries/index.cfm
(2) http://data.worldbank.org/indicator/SH.DYN.AIDS.ZS
(3) Langkamp, G and Hull, J. *Quantitative Reasoning and the Environment: Mathematical Modeling in Context*. 2007.